Practical Amplifier

To analyse the circuit:
- Determine quiescent conditions
- Calculate mutual conductance
- Calculate small signal performance
  - Voltage Gain
  - Input Impedance
  - Output Impedance
  - Cut-off frequency
Quiescent Conditions

\[ I_B \approx 0 \Rightarrow I_{RB} \approx 0 \Rightarrow V_B \approx 0 \]

\[ V_{BE} \approx 0.5 \text{ V} \Rightarrow V_E \approx -0.5 \text{ V} \]

\[ I_E = I_{RE} = \frac{V_E - (-15)}{R_E} \]

\[ = \frac{14.5}{29} = 0.5 \text{ mA} \approx I_C \]

\[ V_C = 15 - I_C R_C \]

\[ = 15 - 0.5 \times 10 = 10 \text{ V} \]
Small Signal Analysis: Voltage Gain

As before:

\[ i_C = g_m v_{BE} = g_m v_{IN} \]

\[ \frac{v_{OUT}}{i_C} = \frac{dV_{OUT}}{dI_C} = -R_C \]

\[ \frac{v_{OUT}}{v_{IN}} = \frac{i_C}{i_C} \times \frac{i_C}{v_{IN}} = -R_C g_m \]

\[ = -R_C \times \frac{I_C}{V_T} = -10^5 \times \frac{0.5}{25} = -200 \]
Input and Output Impedance

- Unlike the op-amp, transistor amplifiers have significant output impedances and finite input impedances
  - $R_{IN}$ can be comparable with the source resistance of the input signal
  - $R_{OUT}$ can be comparable with the load resistance

![Circuit Diagram]
Input Impedance

- Input impedance, $r_{IN}$, is the ratio of the small signal input voltage and the small signal input current.

$$r_{IN} = \frac{v_{IN}}{i_{IN}}$$

$$i_{IN} = i_{RB} + i_B$$

$$i_{RB} = \frac{v_{IN}}{R_B}$$

$$i_B = \frac{i_C}{\beta} = \frac{v_{IN} g_m}{\beta}$$
Input Impedance (cont)

\[i_{IN} = i_{RB} + i_B = \frac{v_{IN}}{R_B} + \frac{v_{IN} g_m}{\beta}\]

\[r_{IN} = \frac{v_{IN}}{i_{IN}} = \frac{1}{1/R_B + g_m / \beta} = R_B \parallel \frac{\beta}{g_m}\]

NB. \( g_m = \frac{i_C}{v_{BE}} \) \& \( r_E = \frac{v_{BE}}{i_E} \)

\[\therefore g_m \approx \frac{1}{r_E}\]
Output Impedance

- One way to measure $r_{OUT}$ is:
  - Short the input to 0 V
  - Output now looks like just $r_{OUT}$

\[ v_{IN} = 0 \]
\[ r_{IN} \quad A v_{IN} \quad r_{OUT} \quad v_{OUT} \]
\[ \equiv \quad r_{OUT} \quad v_{OUT} \]
Output Impedance (cont)

$v_{IN} = 0 \Rightarrow i_C = 0$

Applying Kirchoff’s current law:

$i_C = i_{RC} + i_{OUT} = 0 \Rightarrow i_{OUT} = -i_{RC}$

By Ohm’s law:

$V_C = 15 - I_{RC}R_C \Rightarrow \frac{v_C}{i_{RC}} = -R_C = \frac{v_{OUT}}{i_{RC}}$

$r_{OUT} = \frac{v_{OUT}}{i_{OUT}} = -\frac{v_{OUT}}{i_{RC}} = -(-R_C) = R_C$
Coupling Capacitors

- Capacitor $C_{OUT}$ is needed to remove the d.c. component of the collector voltage.
- Capacitor $C_{IN}$ is needed to allow the base voltage to be offset from 0V.
- In both cases this is known as coupling.
- Both capacitors are chosen to look like short circuits at operating frequencies.
- Their reactance will, however, become significant at low frequencies.

![Diagram of a coupling capacitor circuit](image-url)
Equivalent Circuit

\[ v_1 = v_{IN} \left( \frac{r_{IN}}{r_{IN} + 1/j2\pi f C_{IN}} \right) = v_{IN} \left( \frac{1}{1 + 1/j2\pi f r_{IN} C_{IN}} \right) \]

\[ |v_1| = |v_{IN}| \left( \frac{1}{\sqrt{1 + 1/(2\pi f r_{IN} C_{IN})^2}} \right) \]
Cut-Off Frequency

Cut-off frequency, or –3dB point, is when the gain of the amplifier falls by a factor of $\sqrt{2}$

$$\frac{|v_1|}{|v_{IN}|} = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1+1/(2\pi f_C r_{IN} C_{IN})^2}}$$

$$\Rightarrow 1 + 1/(2\pi f_C r_{IN} C_{IN})^2 = 2 \Rightarrow 2\pi f_C r_{IN} C_{IN} = 1$$

If the cut-off frequency, $f_C$ is specified and $r_{IN}$ has been calculated:

$$C_{IN} = \frac{1}{2\pi f_C r_{IN}}$$

NB. This assumes that $C_{OUT}$ still looks like a short circuit
For the lower cut-off frequency calculation to be valid, $C_{\text{OUT}}$ should still look like a short circuit at $f_c$

$$\Rightarrow X_c = \frac{1}{2\pi f_c C_{\text{OUT}}} \ll r_{\text{OUT}}$$

Typically, choose:

$$C_{\text{OUT}} \geq \frac{1}{2\pi \frac{f_c}{10} r_{\text{OUT}}}$$
For the highest voltage gain,

\[ v_{BE} \approx v_{IN} \Rightarrow v_{BE} \gg v_E \]

But,

\[ v_{BE} = \frac{i_C}{g_m} = i_E r_E \]

where,

\[ r_E \approx \frac{1}{g_m} = \frac{V_T}{I_C} \]

Also,

\[ v_E = \frac{i_E}{j2\pi f C_E} \]

\[ \therefore v_E j2\pi f C_E = \frac{v_{BE}}{r_E} \]
Emitter Capacitor (cont)

For $C_E$ to not interfere at $f_C$:

$$v_{BE} \gg v_E$$

Where,

$$v_E j 2\pi f_C C_E = \frac{v_{BE}}{r_E}$$

$$\Rightarrow 2\pi f_C C_E r_E \ll 1$$

To make sure, choose,

$$C_E \geq \frac{1}{2\pi \frac{f_C}{10} r_E}$$

NB. Use $r_E (= V_T/I_C)$ not $R_E$ for this calculation!
In the context of the common-emitter amplifier we have covered:
- Small signal analysis
- Mutual conductance
- Input/output impedance
- Coupling capacitor requirements and cut-off frequencies

Next time:
- Applying the same principles to the differential amplifier
- It’s actually a much easier circuit to analyse – honest!
- Make sure you’re happy with the fundamentals by then!